Portfolio Strategy Analysis Based on USD, Bitcoin, and Gold

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Abstract: In this paper, we build time-series forecasting models and goal-planning models for gold and bitcoin and provide optimal gold and bitcoin rotation investment strategies based on our approach. First, we used a line graph to compare the SVM-GARCH predictions with the actual data provided by the subjects and found that the predictions were highly consistent with the actual scene. Using four metrics, MSE, RMSE, MAE, and MAPE, we demonstrate that our combined model has higher prediction accuracy than a single model. Secondly, the sensitivity analysis of the planning model was carried out using the gold and bitcoin transaction commission rates. As the gold and bitcoin transaction commission rates increased, the transaction share under the optimal combination strategy decreased, which proved the rationality of the planning model.

1. Introduction

Nifty's new show "The Squid Game" has caught fire worldwide, becoming one of the most-played series. This game of life and death has resonated in the asset markets: although gold and bitcoin belong to two different asset classes with different risk appetites, since they both operate in large pools of money, they are inevitably drawn together and participate in the "squid game" chosen by investors [1]. Since 2016, annual bitcoin production has been halved, black swan events such as Brexit have occurred, Asian financial markets have seen an "asset shortage," investors have included digital currencies in the underlying category, global financial markets have been volatile, gold has become the preferred safe haven for investors, and the bitcoin market has seen an upswing. We may not know who is the survivor of this "squid game," but the organic combination of gold and bitcoin may lead to a different kind of "surprise."

This work proposes a prediction model for the SVM-GARCH price return time series based on nearest neighbor mutual information feature selection, which is used to make day-by-day rolling forecasts by fixing the reference period. We organically combine investment returns and risks through the VaRY model [2], thus transforming multi-objective planning into single-objective planning. Finally, we demonstrate the superiority of our model to investors in four different dimensions by juxtaposing evidence and analysis through prediction accuracy, planning rationality, model resilience, and investment outcome analysis.

2. SVM-GARCH Model

We introduce the concept of neighborhood mutual information and construct the SVM-GARCH model. The correlation data of gold and bitcoin price trends are extracted using the neighboring mutual information, SVM extracts the nonlinear components in gold and bitcoin price trends, and the heteroskedasticity of the forecast residuals is handled by the GARCH model while considering the stock price volatility.

It is supported vector machine regression can be represented as an optimization problem of the following form:

$$\min_{w,b,\xi} \frac{1}{2}\|w\|^2 + C\sum_{i=1}^{N}(\xi_i + \xi_i^*) \tag{1}$$

$$\text{s.t.}\begin{cases} y_i - < w, x_i > -b \le \varepsilon + \xi_i \\ < w, x_i > +b \le \varepsilon + \xi_i^* \\ \xi_i, \xi_i^* \ge 0 \end{cases} \tag{2}$$

Where ξ_i and ξ_i^* is the relaxation factor and C is the penalty parameter. The above optimization problem can be expressed as a dyadic form:

$$\max_{\alpha} \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}(\alpha_i^* - \alpha_i)\left(\alpha_j^* - \alpha_j\right)K(x_i - y_i) + \varepsilon\sum_{i=1}^{N}(\alpha_i^* + \alpha_i) - \sum_{i=1}^{N} y_i(\alpha_i^* - \alpha_i) \tag{3}$$

$$\text{s.t.}\begin{cases} \sum_{i=1}^{N}(\alpha_i^* - \alpha_i) = 0 \\ 0 \le \alpha_i, \alpha_i^* \le \frac{C}{N}, i = 1,2, \ldots, N \end{cases} \tag{4}$$

We can obtain the prediction $f(x)$ when the input is x:

$$f(x) = \sum_{i=1}^{I}(\alpha_i - \alpha_i^*)\, K(x, x_i) - b \tag{5}$$

In Equation (5), α_i, α_i^* is the Lagrange multiplier, $K(x, x_i)$ is the kernel function.

Assuming that r_t is the return time series data, the mean and variance equations of the SVM-GARCH model are as follows:

$$r_t = c + \emptyset f(x_t) + \sum_{j=1}^{q}\theta_j\, u_{t-j} + u_t \tag{6}$$

$$\sigma^2 = \alpha_0 + \sum_{i=1}^{r}\alpha_i\, u_{t-i}^2 + \sum_{j=1}^{s}\beta_j\, \sigma_{t-j}^2 \tag{7}$$

Where q, r, and s denote the lag order, and θ_j, α_i, β_j is the lag term parameter. In the mean value equation of the above SVM-GARCH model, $f(x) = \sum_{i=1}^{I}(\alpha_i - \alpha_i^*)\, K(x, x_i) - b$ is the predicted value of SVM and u_{t-j} is the error term of (1) at the moment of $t - j$. The result contains both the past return data and the high-dimensional information related to the return.

If the sample set $U = \{x_1, x_2, \ldots, x_n\}$ is described by the discrete numerical feature set F, R, S is the feature subset of the feature set F, i.e., $R, S \subseteq F$, and the nearest neighbor domain of the sample x_i on the feature subsets, R and S can be denoted as $\delta_R(x_i)$ and $\delta_S(x_i)$, respectively, then the nearest neighbor mutual information of R and S is defined as:

$$NMI_\delta = -\frac{1}{n}\sum_{i=1}^{n} log \frac{\|\delta_R(x_i)\| \cdot \|\delta_S(x_i)\|}{n\|\delta_{SUR}(x_i)\|} \tag{8}$$

The concept of nearest-neighbor mutual information not only satisfies the need to express the nonlinear relationship between the time series of returns but also solves the difficulty of calculating the associated edge probability density and joint probability density of the traditional mutual information in computing the mutual information of discrete numerical data [4].

The general idea of the SVM-GARCH price return time series forecasting model based on the nearest neighbor mutual information feature selection is as follows: firstly, we use the nearest neighbor mutual information to select the historical data of the target market with a strong correlation with the target market return and the surrounding market information to construct the high-dimensional input variable information for the support vector machine regression; then we train the SVM analysis to process the return time series data; Finally, a GARCH model is used to analyze the heteroskedasticity of the residual series to correct and improve the validity and accuracy of the SVM-GARCH model prediction.

$x_i^* = (x_{i,1}^*, x_{i,2}^*, \ldots, x_{i,k}^*)$ is the k-dimensional input variable containing the previous P-period return data and the information that the K-P dimension strongly correlates with the gold (or bitcoin) return correlation.

3. Model Improvements

3.1 Dynamic weighted multi-objective planning model

In this paper, the VaRY model combines the two, thus transforming multi-objective planning into single-objective planning, and the risk adjustment factor V is used as the dynamic weight adjustment basis of the model to accommodate the differences in utility tendencies of different investors.

3.2 Trading Day Decision Model

3.2.1 Risk Quantification

Both Bitcoin and gold can trade during the trading day, so we measure the portfolio risk of Bitcoin and gold using the VaR calculation for the asset portfolio case [3], which is calculated as follows.

Assuming that the single-period return of the i asset is $r_i, i = 1,2$ and w_i is the weight of the i asset, the return, and variance of the portfolio are:

$$R_p = \sum_{i=1}^{2} w_i \cdot r_i = w^T \cdot r \tag{9}$$

$$\sigma_p^2 = \sum_{i=1}^{2} w_i^2 \cdot \sigma_{ii} + \sum_{i=1}^{2} \sum_{j=1, i \neq j}^{2} w_i w_j \rho_{ij} \sigma_i \sigma_j = \sum_{i=1}^{2} w_i^2 \, \sigma_i^2 + 2 \sum_{i=1}^{2} \sum_{j<i}^{2} w_i w_j \, \sigma_{ij} = w^T \sum w \tag{10}$$

Where σ_{ii} is the variance of the return of the asset i , ρ_{ij} is the correlation coefficient between the returns of assets i and j(j = 1,2), $\rho_{ij}\sigma_i\sigma_j = \sigma_{ij}$,$\sum$ is the variance-covariance matrix, $\sum = [\sigma_{ij}]$, where w denotes the weight vector and r denotes the return vector of the asset. If the returns of each asset obey a normal distribution, then the portfolio returns also obey a normal distribution, at which point we have:

$$VAR_{portfolio} = V_{t-1} Z_a \sqrt{w^T \sum w} \tag{11}$$

where $V_{t-1} = \sum_{i=1}^{2} w_i \, V_{t-1,i}$, $V_{t-1,i}$ is the yesterday's closing price of the asset i , and $C_{ov}(V_{Gold}, V_{Bitcoin})$ is the covariance calculated by using the last 7 days of price data as a sample.

3.2.2 Wealth Utility

We convert the dual-objective planning model to a single-objective planning model by organically combining return and risk through the VaRY mode [5]l, and by hedging the positive benefits from return with the negative benefits from risk to find the optimal weights of gold, bitcoin w_g, w_b.

$$\text{maxVARY}_{portfolio} = U_{positive}(Yield) - U_{negative}(VaR) \tag{12}$$

$$Yield = \frac{Closing\ Price - Opening\ Price}{Opening\ Price} \tag{13}$$

3.2.3 Profit and Loss Selection

According to the profit and loss status of different purposes, the final weights for judging Pre and outputting the optimal investment decision are established, as shown in Table 1.

Table 1 Profit and Loss Basis

$Pre_{portfolio} = \Delta w_{t,gold}(Yield_{gold} - \alpha_{gold}) + \Delta w_{t,bitcoin}(Yield_{bitcoin} - \alpha_{bitcoin})$	
Judgment Basis	Profit and Loss Status
$Pre_{portfolio} \geq 0$	Profit
$Pre_{portfolio} < 0$	Loss

Where U is the profit margin for gold and bitcoin and α_{gold} represents the commission rate for gold and bitcoin.

The optimal weighting loss should be compared to the minimum loss of the selling strategy, which is calculated as follows:

$$\lambda = w_{gold} \cdot Deficit_{t+1,gold} + (1 - w_{gold}) w_{gold} \cdot Deficit_{t+1,bitcoin} \tag{14}$$

where λ is the fixed loss amount under the optimal weight of VaRY, Deficit that $|Yield|$ is under the $Closing\ Price < Opening\ Price$, x, y is the respective selling weight of gold and bitcoin under the optimal selling strategy (minimum loss).

The trading day operation flow is shown in Fig.1.

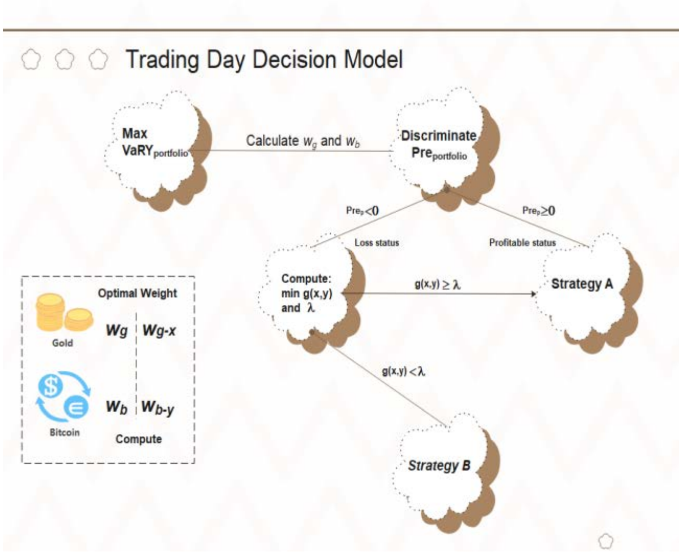

Figure 1 Flowchart of investment decision sketch

3.3 Non-trading day model

The VaR calculation for the single-asset scenario is used herein to measure the investment risk of Bitcoin, which is calculated as follows:

$$\mathrm{VaR}_{single} = Z_a\sqrt{\left(1-w_{t,gold}\right)^2\sigma_{t,bitcoin}^i}\cdot\sqrt{T} = V_{t-1}\cdot\frac{u-r^*}{\sigma}\cdot\sqrt{\left(1-w_{t,gold}\right)^2\sigma_{t,bitcoin}^i}\cdot\sqrt{T} \quad (15)$$

where V_{t-1} denotes the yesterday's closing price of bitcoin, where r denotes the expected return of the asset over the holding period T, r^* denotes the minimum return corresponding to the confidence level α, i.e., the lowerα quantile of the return.

We construct wqe for this asset [6], bitcoin, to determine the optimal daily holding weight of bitcoin during non-trading days, and the calculation process is as follows:

$$maxVaRY_{portfolio} = U_{positive}(Yield) - U_{nagative}(VaR) = v_2\left(w_{t,bitcoin}Yield_{itcoin}\right) - v_1\cdot VaR \quad (16)$$

Adjust the weights established by VaRY according to the different objectives in the profit or loss state, as shown in Table 2

Table 2 Profit and loss selection

<table>
<tr><td colspan="2">$$Pre_{single} = \sum_i^N \Delta w_{b,i}\left(Yield_{bitcoin,i} - \alpha_{bitcoin}\right) - \max\{\max\{w_{b,1}, w_{b,2}, \ldots, w_{b,N}\} - w_{g,i-1}, 0\} \cdot \alpha_{gold}$$</td></tr>
<tr><td>Judgment Basis</td><td>Profit and Loss Status</td></tr>
<tr><td>$Pre_{portfolio} \geq 0$</td><td>Profit</td></tr>
<tr><td>$Pre_{portfolio} < 0$</td><td>Loss</td></tr>
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The non-trading day decision model first combines and hedges the risk and return of investment through the VaRY model and uses the risk adjustment factor as the dynamic weight adjustment basis to determine the optimal investment weights for investors with different risk preferences shown in Fig.2.

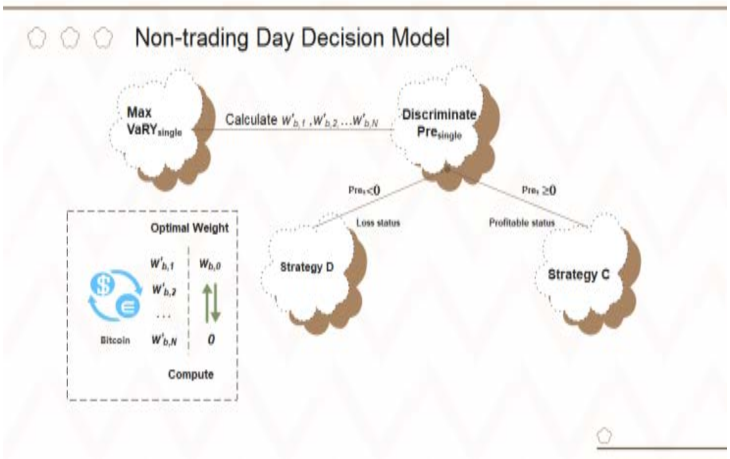

Figure 2 Flowchart of investment decision sketch

4. Analysis Process

We use line graphs to compare the forecasting results of the SVM-GARCH model with those of the SVM model to demonstrate the model's excellence visually. Also, we demonstrate that our combined model has higher forecasting accuracy than a single model by using five indicators: MSE, RMSE, MAE, MAPE, and R2.

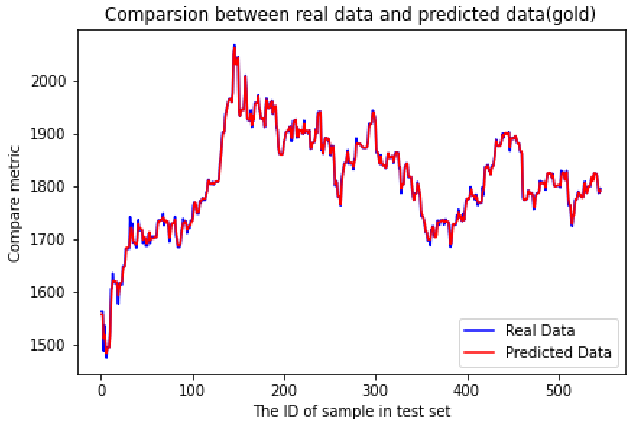

Figure 3 Gold forecast price comparison chart

In Fig.3, the blue line represents the actual value of gold price, while the green line represents the predicted value of gold price. As we can see, the curve predicted by our model is almost the same as the curve drawn by the actual value of gold price, which indicates that our model has good prediction accuracy.

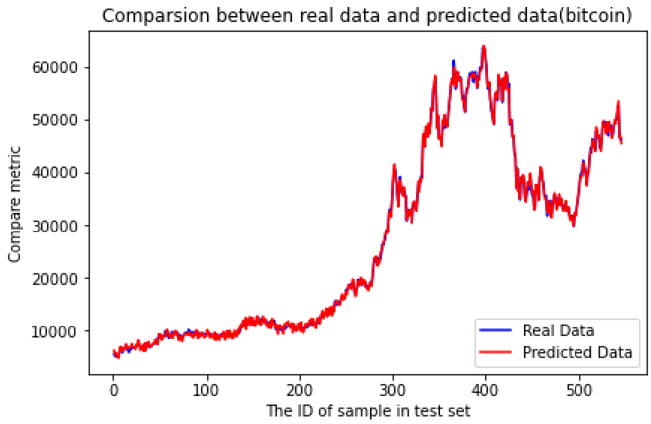

Figure 4 Bitcoin forecast price comparison chart

In Fig.4, the blue line represents the actual value of the bitcoin price, while the green line represents the predicted value of the bitcoin price. It can be seen that the curve predicted by our model is roughly consistent with the curve drawn from the actual value of the gold price, with slight differences, which indicates that our model has a good prediction accuracy.

In order to reasonably analyze the sensitivity of our investment model to transaction costs, we control the other parameters constant and further observe the changes in the model results by changing the commission rate of bitcoin or gold in the model.

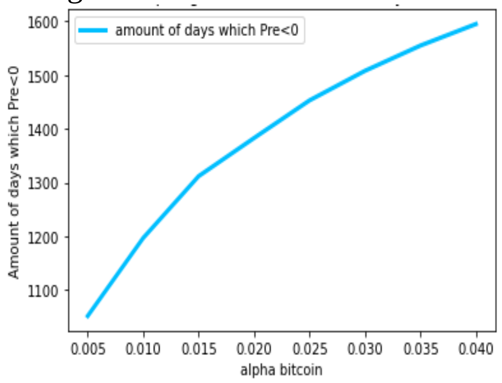

Figure 5 Control α_{gold}=1% to keep it constant

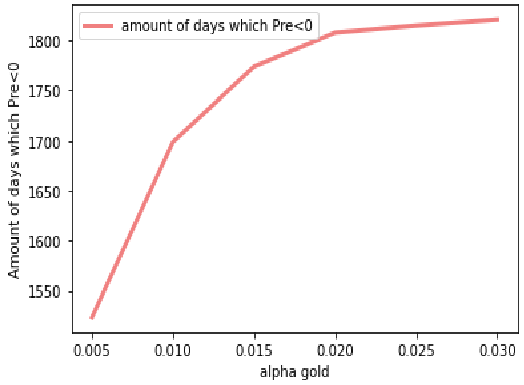

Figure 6 Control $\boldsymbol{\alpha_{Bitcoin}}$=2% to keep it constant

In Fig. 5, this may be due to the consideration of investment efficiency, when the cost rises, investors tend to be more worried about the damage to earnings, so that the awareness of avoiding the emergence of losses increased, so the number of days (Pre < 0) increases at a slower rate, which is in line with the reality.

In Fig. 6, it is worth noting that the number of days (Pre < 0) smoothes out faster, suggesting that our model has a more sustained sensitivity to bitcoin's commission rate fluctuations, which may be due to bitcoin's high-return and high-risk nature. Its more enormous cost changes may bring greater utility fluctuations for investors, showing that our model is practically meaningful.

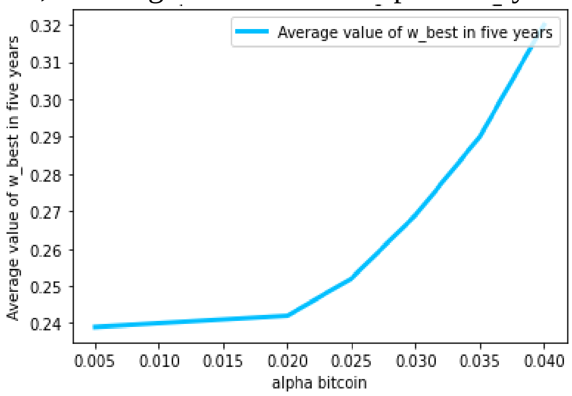

Figure 7 Control for α_{gold}= 1% to keep constant

Fig. 7 indicates that the cost change of bitcoin has a significant negative impact on its return utility at this time, probably because the cost of bitcoin continues to rise. The ability of gold to resist risk becomes more important to investors, which in turn causes the holding weight of bitcoin to fall rapidly, thus indirectly causing the gold weighting to rise more quickly, which is realistic.

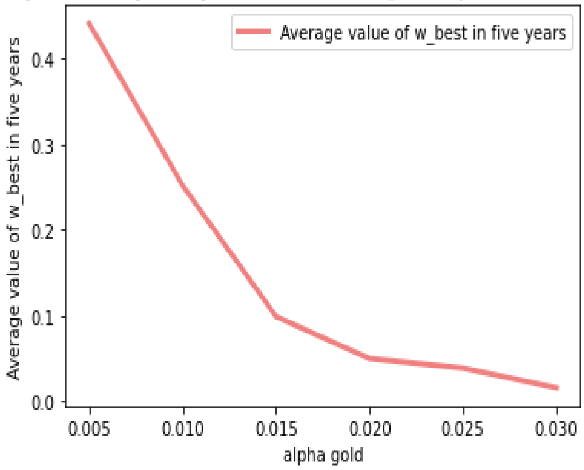

Figure 8 Control for $\boldsymbol{\alpha_{Bitcoin}}$=2% to keep constant

Fig.8 indicates that for gold, which has a more balanced return utility and risk-utility, a smaller change in cost will cause a corresponding decrease in the weight of gold holdings. In the (2.1%, 3%) range, as the α_{gold} rises, the sensitivity of gold weighting decreases, which may be due to gold's ability to withstand risk so that investors often have to hold a certain amount of gold to hedge against the high risk brought by bitcoin, so as the cost rises, the degree of decline in the weight of gold holdings will gradually slow down, which is also in line with the actual.

5. Conclusion

In this paper, in order to obtain optimal portfolio strategies for gold and bitcoin, a dynamic weighted multi-objective programming model is constructed based on the time series forecasting model. In terms of investment forecasting, we first introduced nearest neighbor mutual information and constructed the SVM-GARCH combination model. Based on fully considering the correlation between gold and Bitcoin, the data were extracted by nonlinear feature extraction and heteroscedasticity processing to provide a more accurate and reliable basis for price fluctuations' impact on investment decision planning. The VARY model is further constructed to reasonably balance investment returns and investment risks in terms of investment planning. Risk adjustment factors are introduced into the planning model to establish a multi-objective planning model with different dynamic weights for trading and non-trading periods.

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